Class IX Session 2024-25
Subject - Mathematics
Sample Question Paper - 4

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. Ordinate of all points on the y-axis is **[1]**

a) 0 b) -1

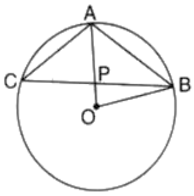
c) any number d) 1

2. If the area of an isosceles right triangle is 8 cm^2, what is the perimeter of the triangle? **[1]**

a) $8 + 4\sqrt{2}$ cm^2 b) $8 + \sqrt{2}$ cm^2

c) $12\sqrt{2}$ cm^2 d) $4 + 8\sqrt{2}$ cm^2

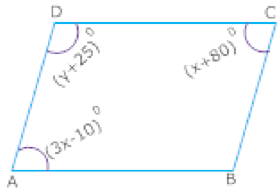
3. In the given, AB is side of regular five sided polygon and AC is a side of a regular six sided polygon inscribed in the circle with centre O. AO and CB intersect at P, then $\angle APB$ is equal to **[1]**



a) 90^o b) 72^o

c) 86^o d) 96^o

4. In the fig, ABCD is a Parallelogram. The values of x and y are **[1]**



a) 45°, 30°

b) 30°, 35°

c) 45°, 45°

d) 55°, 35°

5. The value of 1.9999……………… in the form $\frac{p}{q}$, where 'p' and 'q' are integers and q $\neq$ 0, is **[1]**

a) $\frac{1999}{1000}$

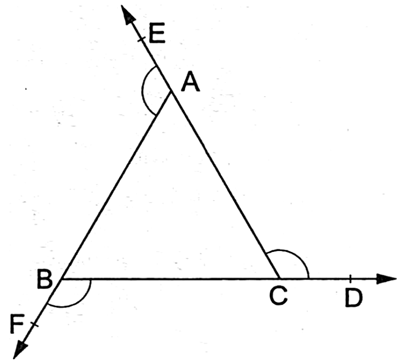
b) $\frac{19}{10}$

c) 2

d) $\frac{1}{9}$

6. The sides BC, CA and AB of $\triangle$ABC have been produced to D, E and F respectively. **[1]**

$\angle BAE + \angle CBF + \angle ACD =$?



a) 240°

b) 360°

c) 300°

d) 320°

7. **The cost of a notebook is twice the cost of a pen.** The equation to represent this statement is **[1]**

a) 2x = 3y

b) x = 3y

c) x + 2y = 0

d) x - 2y = 0

8. The zeros of the polynomial p(x) = $3x^2$ - 1 are **[1]**

a) $\frac{1}{3}$ and 3

b) $\frac{1}{\sqrt{3}}$ and $\frac{-1}{\sqrt{3}}$

c) $\frac{-1}{\sqrt{3}}$ and $\sqrt{3}$

d) $\frac{1}{\sqrt{3}}$ and $\sqrt{3}$

9. The value of $\left(x^{a-b}\right)^{a+b} \times \left(x^{b-c}\right)^{b+c} \times \left(x^{c-a}\right)^{c+a}$ is **[1]**

a) 3

b) 2

c) 1

d) 0

10. If a diagonal AC and BD of a quadrilateral ABCD bisect each other, then ABCD is a **[1]**

a) Parallelogram

b) Rhombus

c) Rectangle

d) Triangle

11. The value of $\sqrt{3 - 2\sqrt{2}}$ is **[1]**

a) $\sqrt{2} + \sqrt{1}$

b) $\sqrt{2} - \sqrt{1}$

c) $\sqrt{3} + \sqrt{2}$

d) $\sqrt{3} - \sqrt{2}$

12. x = 2, y = 5 is a solution of the linear equation **[1]**

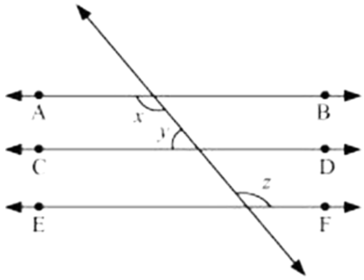
a) 5 x + y = 7

b) x + y = 7

c) 5x +2y = 7

d) x + 2y = 7

13. In the given figure, AB || CD, CD || EF and y : z = 3 : 7, then x =? **[1]**



a) 63°

b) 126°

c) 108°

d) 162°

14. If x = $\sqrt{5}$+ 2, then x - $\frac{1}{x}$ equals **[1]**

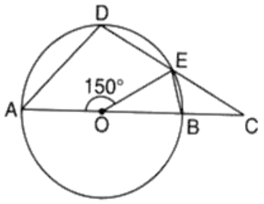
a) 2

b) 4

c) $2\sqrt{5}$

d) $\sqrt{5}$

15. AOB is the diameter of the circle. If $\angle AOE = 150^o$, then the measure of $\angle CBE$ is **[1]**



a) 115^o

b) 125^o

c) 120^o

d) 105^o

16. A point of the form $(0, b)$ lies on: **[1]**

a) x- axis

b) quadrant I

c) quadrant III

d) y- axis

17. If a linear equation has solutions (-2, 2), (0, 0) and (2, -2), then it is of the form: **[1]**

a) x + y = 0

b) -2x + y = 0

c) x – y = 0

d) -x + 2y = 0

18. The maximum number of zeroes that a polynomial of degree 3 can have is **[1]**

a) Zero

b) One

c) Two

d) Three

19. **Assertion (A):** In ΔABC, E and F are the midpoints of AC and AB respectively. The altitude AP at BC intersects FE at Q. Then, AQ = QP. **[1]**

Reason (R): Q is the midpoint of AP.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** 5 - $\sqrt{2}$ = 5 - 1.414 = 3.586 is an irrational number. **[1]**

Reason (R): The difference of a rational number and an irrational number is an irrational number.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

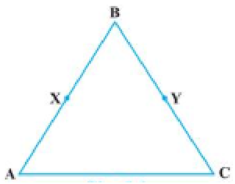
c) A is true but R is false.

d) A is false but R is true.

Section B

21. Point C is called a mid point of line segment AB, prove that every line segment has one and only one mid-point. **[2]**

22. In the given figure, we have $BX = \frac{1}{2}AB$ and $BY = \frac{1}{2}BC$ and AB = BC. Show that BX = BY. **[2]**



23. Name the quadrant in which the point lies :(i) A(1, 1) (ii) (–2, –4) (iii) C(1, –2). **[2]**

24. Find two rational and two irrational numbers between 0.5 and 0.55. **[2]**

OR

Express the decimal $0.\overline{235}$ in the form $\frac{p}{q}$, where p, q are integers and q $\neq$ 0.

25. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find the radius of the base. **[2]**

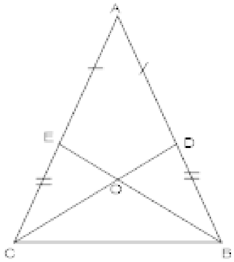
OR

If the radius and slant height of a cone are in the ratio 7 : 13 and its curved surface area is 286 cm^2, find its radius.

Section C

26. Find the value of $\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$ **[3]**

27. If AE = AD and BD = CE. Prove that $\triangle AEB \cong \triangle ADC$ **[3]**

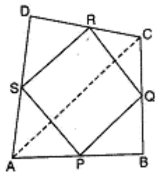


28. ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA. AC is a diagonal. Show that **[3]**

i. SR || AC and SR = $\frac{1}{2}$AC

ii. PQ = SR

iii. PQRS is a parallelogram.

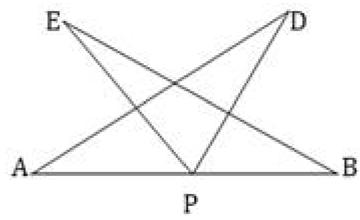


29. Find at least 3 solutions for the linear equation 2x – 3y + 7 = 0. **[3]**

30. AB is a line segment and P is the mid-point. D and E are points on the same side of AB such that $\angle$ BAD = $\angle$ ABE and $\angle$ EPA = $\angle$ DPB. Show that: **[3]**

i. $\Delta DAP \cong \Delta EBP$

ii. AD = BE (See figure)



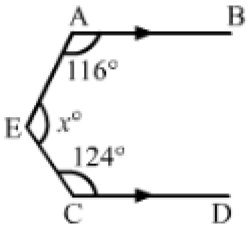
OR

ABC is an isosceles triangle in which AB = AC. BE and CF are its two medians. Show that BE = CF.

31. If x + y + z = 0, show that $x^3 + y^3 + z^3 = 3xyz$. **[3]**

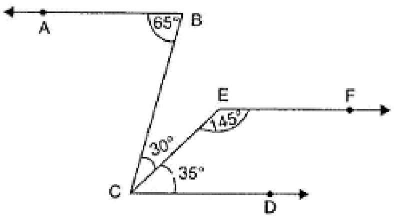
Section D

32. In each of the figures given below, AB || CD. Find the value of x° in each other case. **[5]**



OR

In figure, $\angle ABC = 65^o$, $\angle BCE = 30^o$, $\angle DCE = 35^o$ and $\angle CFE = 145^o$. Prove that AB || EF.



33. Prove that the line joining the mid-points of the diagonals of a trapezium is parallel to each of the parallel sides and is equal to half of the difference of these sides. **[5]**

34. The perimeter of a triangular field is 420 m and its sides are in the ratio 6 : 7 : 8. Find the area of the triangular field. **[5]**

OR

The perimeter of a right triangle is 24 cm. If its hypotenuse is 10 cm, find the other two sides. Find its area by using the formula area of a right triangle. Verify your result by using Heron's formula.

35. Factorize: $x^3 - 2x^2 - x + 2$ **[5]**

Section E

36. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and a height of 1 m. **[4]**

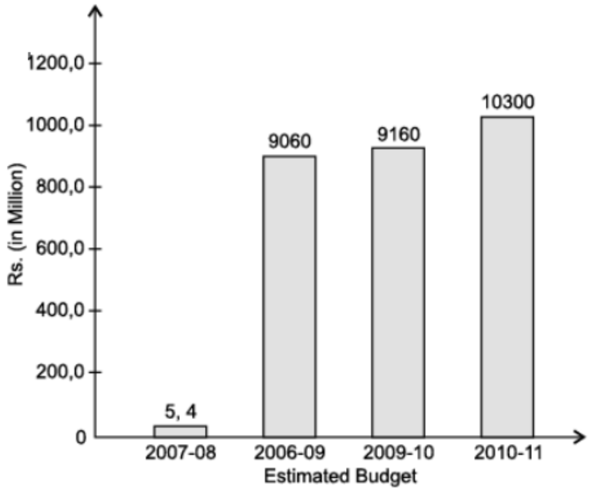
i. Find the curved surface area of the cone.

ii. If the outer side of each of the cones is to be painted and the cost of painting is ₹ 12 per m^2, what will be the cost of painting all these cones? (Use π = 3.14 and take $\sqrt{1.04}$ = 1.02)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Ladli Scheme was launched by the Delhi Government in the year 2008. This scheme helps to make women strong and will empower a girl child. This scheme was started in 2008.

The expenses for the scheme are plotted in the following bar chart.



i. What are the total expenses from 2009 to 2011? (1)

ii. What is the percentage of no of expenses in 2009-10 over the expenses in 2010-11? (1)

iii. What is the percentage of minimum expenses over the maximum expenses in the period 2007-2011? (2)

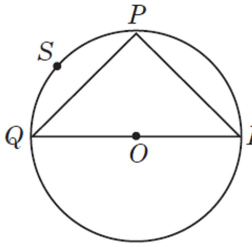
OR

What is the difference of expenses in 2010-11 and the expenses in 2006-09? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Sanjay and his mother visited in a mall. He observes that three shops are situated at P, Q, R as shown in the figure from where they have to purchase things according to their need. Distance between shop P and Q is 8 m and between shop P and R is 6 m.

Considering O as the center of the circles.



i. Find the Measure of $\angle QPR$. (1)

ii. Find the radius of the circle. (1)

iii. Find the Measure of $\angle QSR$. (2)

OR

Find the area of ΔPQR. (2)

Solution

Section A

1.

(c) any number

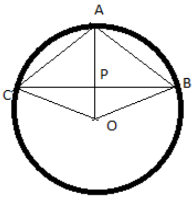
Explanation: In the cartesian plane any point P is written as p(x, y)
when the value of x co-ordinate is equal to zero then the point P lies on y axis,
So,Ordinate of any point on y-axis can be any number but abscissa will be zero

2. **(a)** $8 + 4\sqrt{2}$ cm^2

Explanation: Let each of the two equal sides of an isosceles right triangle be a cm
Then, third side = $a\sqrt{2}$m
Area of $\Delta = \frac{1}{2} \times 2 \times 2$
$\Rightarrow 8 = \frac{a^2}{2}$
$\Rightarrow$ $a^2 = 16$
$\Rightarrow$ a = 4 cm
$\Rightarrow$ Perimeter
$\Rightarrow$ a + a + a $\sqrt{2}$ = 4 + 4 + 4 $\sqrt{2}$ = 8 + 4$\sqrt{2}$ cm^2

3.

(d) 96^o

Explanation: 

Here AC is side of hexagon, so it will subtend 60° angle at centre and also sides & radius are equal.
Thus, AC = OC = OA and $\angle COA = \angle OAC = \angle ACO$ = 60°
AB is side of pentagon, so it would subtend angle of $\frac{360}{5}$ = 72° angle at centre.
so, $\angle BOP$ = 72°
SO, $\angle COB = 72 + 60 = 132^\circ$
Also since, OC = OB, $\angle OCP = OBP$
$\triangle COB$ $\angle COB + \angle OBC + \angle OCB$ = 180°
2$\angle OBC$ = 180 - (132) = 48°
$\angle OBC$ =24°
NOW, $\triangle$ BOP $\angle BOP +$ $\angle OPB +$ $\angle PBO$ =180°
$\angle OPB$ = 180 - (24 + 72) =180 - 96 = 84°
Now,
$\angle APB$ and $\angle OPB$ lie on straight line, so they are supplementry angles.
$\angle APB$ = 180 - $\angle OPB$ = 180- 84 =96°

4. **(a)** 45°, 30°

Explanation: 3x - 10° = x + 80° [opposite angles of a parallelogram are equal.];
3x - x = 80° + 10°;
2x = 90°;
x = 90°/2;
x = 45°
3x - 10° + y +25° = 180° [In a parallelogram co-interior angles are supplementary.];
3 $\times$ 45° - 10° + y + 25° = 180°;
135° + 25° - 10° + y = 180°;

$150° + y = 180°$;

$y = 180° - 150° = 30°$

5.

(c) 2

Explanation: 1.9999 can be written as 2,

2is taken as approx vlaue .

6.

(b) 360^{o}

Explanation: We have :

$\angle 1+\angle BAE=180^{\circ}$ ....(i)

$\angle 2+\angle CBF=180^{\circ}$ ......(ii)

$\angle 3+\angle ACD=180^{\circ}$ .....(iv)

Adding (i),(ii) and (iii), we get:

$(\angle 1+\angle 2+\angle 3)+(\angle BAE+\angle CBF+\angle ACD)=540^{\circ}$

$\Rightarrow 180^{\circ}+\angle BAE+\angle CBF+\angle ACD=540^{\circ}$ $[\because \angle 1+\angle 2+\angle 3=180^{\circ}]$

$\Rightarrow \angle BAE+\angle CBF+\angle ACD=360^{\circ}$.

7.

(d) x - 2y = 0

Explanation: Let the cost of the notebook is ₹ x and pen is ₹ y and we have given that the cost of a notebook is twice the cost of a pen.

So we have

x = 2y

or x - 2y = 0

8.

(b) $\frac{1}{\sqrt{3}}$ and $\frac{-1}{\sqrt{3}}$

Explanation: Let: $p(x) = 3x^2 - 1$

To find the zeroes of p(x), we have:

$p(x) = 0 \Rightarrow 3x^2 - 1 = 0$

$\Rightarrow 3x^2 = 1$

$\Rightarrow x^2 = \frac{1}{3}$

$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow x = \frac{1}{\sqrt{3}}$ and $x = \frac{-1}{\sqrt{3}}$

9.

(c) 1

Explanation: $\left(x^{a-b}\right)^{a+b} \times \left(x^{b-c}\right)^{b+c} \times \left(x^{c-a}\right)^{c+a}$

$\Rightarrow x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$

$\Rightarrow x^{a^2-b^2+b^2-c^2+c^2-a^2}$

$\Rightarrow x^0 = 1$

10. **(a)** Parallelogram

Explanation: Two diagonals of quadrilateral form four triangles. Out of these four triangles two triangles of opposite to each other are congruent by SAS. By using CPCT property we can prove that both pair of opposite sides in a quadrilateral are parallel. A quadrilateral with both pair of opposite sides parallel is called parallelogram.

11.

(b) $\sqrt{2}-\sqrt{1}$

Explanation: $\sqrt{3-2\sqrt{2}}$

$= \sqrt{(\sqrt{2})^{2}+(1)^{2}-2\times\sqrt{2}\times 1}$

$= \sqrt{(\sqrt{2}-1)^2}$

$= (\sqrt{2}-1)$

12.

(b) x + y = 7

Explanation: x = 2 and y = 5 satisfy the given equation.

13.

(b) 126°

Explanation: y : z = 3 : 7

Let common ratio be a

y = 3a

z = 7a

x = z (corresponding angle)

x = 7a

x + y =180° (interior angle)

7a + 3a =180°

10 a = 180°

a = 180/10

a = 18

x = 7a

x = 7x 18

x =126°

14.

(b) 4

Explanation: x = $\sqrt{5}$+ 2, then equals

$\frac{1}{x} = \frac{1}{\sqrt{5}+2}$

$= \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2}$

$= \frac{\sqrt{5}-2}{5-4}$

$= \sqrt{5}-2$

now,

x - $\frac{1}{x}$ = $\sqrt{5}$+ 2 - ($\sqrt{5}-2$)

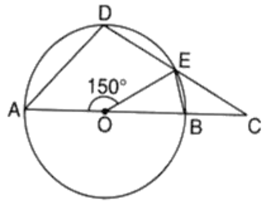
= $\sqrt{5}$ +2- $\sqrt{5}$ + 2

=4

15.

(d) 105^o

Explanation:



Here, AOB is diameter,

so, $\angle BOE$ = 180 - 150 = 30° {Angles lie in straight line}

Now, OE & OB are radius so, OE = OB .i.e$\angle OEB = \angle OBE$

In $\triangle BOE$, $\angle BOE$ + $\angle OBE + \angle BEO$ =180°

= 30 + 2 $\angle OBE$ =180°

= 2 $\angle OBE$ = 180 - 30 = 150°

= $\angle OBE$ = 75°

Now, $\angle OBE$ & $\angle CBE$ lie on staright line

so, $\angle OBE$ + $\angle CBE$ =180°

$\angle CBE$ = 180 - 75 = 105°

16.

(d) y- axis

Explanation: Let P be any point whose co-ordinate be $P(0, b)$

Then, if the value of x-coordinate or abscissa is zero then the point P lies in y-axis.

17. **(a)** x + y = 0

Explanation: Linear equation has solutions (-2, 2), (0, 0) and (2, -2), then the equation will be

x + y = 0

As all the given three points satisfy the given equation

18.

(d) Three

Explanation: The maximum number of zeroes that a polynomial of degree 3 can have is three because the number of zeroes of a polynomial is equals to the degree of that polynomial.

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

In ΔABC, E and F are midpoint of the sides AC and AB respectively.

FE || BC [By mid-point theorem]

Now, in ΔABP, F is mid-point of AB and FQ || BP. Q is mid-point of AP

AQ = QP

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

Section B

21.

Let a line AB have two mid-points, say, C and D. Then

AB = AC + CB = 2AC . . . . (i) . . . [As C is the mid-point of AB]

and AB = AD + DB = 2AD . . . . (ii) [As D is the mid-point of AB]

From equation (i) and (ii)

AC = AD and CB = DB

But this will possible only when D lies on point C. So every line segment has one and only one mid-point.

22. We have AB = BC [Given]

Now, by Euclid's axiom 7, we have things which are halves of the same things are equal to one another.

$\therefore \frac{1}{2}\text{AB} = \frac{1}{2}\text{BC}$

Hence, BX = BY. [$\because$ $\text{BX} = \frac{1}{2}\text{AB}$ and $\text{BY} = \frac{1}{2}\text{BC}$ (Given)]

23. (i) (+, +) are the signs of the co-ordinates of points in the I quadrant.

∴ A(1, 1) lies in the I quadrant.

(ii) (–, –) are the signs of the co-ordinates of points in the III quadrant.

∴ B(–2, –4) lies in the III quadrant.

(iii) (+, –) are the signs of the co-ordinates of points in the IV quadrant.

∴ C(1, –2) lies in the IV quadrant.

24. We know that, 0.5<0.55

Consider x=0.5, y=0.55 and n=2

$d = \frac{y-x}{n+1} = \frac{0.55-0.5}{2+1} = \frac{0.05}{3}$

Two rational and two irrational numbers which lies between 0.5 and 0.55 are x+d and x+2 d

so we get,

$= 0.5+\frac{0.05}{3}$ and $0.5+2\times\frac{0.05}{3}$

$=\frac{1.5+0.05}{3}$ and $\frac{1.5+0.1}{3}$

$=\frac{1.55}{3}$ and $\frac{1.6}{3}$

By division

= 0.51 and 0.53

Two irrational numbers which lies between 0.5 and 0.55 are 0.5151151115 .... And 0.5353553555

OR

Let x = $0.\overline{235}$

i.e. x = 0.235235..… ….(i)

Multiply both sides by 1000, we get

⇒ 1000x = 235.235235……. ….(ii)

On subtracting (i) from (ii), we get

999x = 235

$\Rightarrow x=\frac{235}{999}$

$\therefore 0.\overline{235}=\frac{235}{999}$

25. Slant height of cone = 14 cm

Let radius of circular end of cone be r.

Curved surface area of cone = πrl

308 cm^2 = ($\frac{22}{7}$ × r × 14) cm

⇒ r = $\left(\frac{308}{44}\right)$ cm = 7 cm

Thus, the radius of circular end of the cone is 7 cm.

OR

We are given that,Two ratio in radius and slant height of a cone = 7 : 13

Let radius (r) = 7x

and slant height (1) = 3x

Curved surface area = πrl

$=\frac{22}{7}\times 7x\times 13x=286$

$286x^2$ = 286

$x^2=\frac{286}{286}=1$

$\therefore x=\sqrt{1}=1$

Therefore Radius = 7x = 7 × 1 = 7 cm

Section C

26. We have $\frac{4}{(216)^{-\frac{2}{3}}}+\frac{1}{(256)^{-\frac{3}{4}}}+\frac{2}{(243)^{-\frac{1}{5}}}$

$=4(216)^{\frac{2}{3}}+(256)^{\frac{3}{4}}+2(243)^{\frac{1}{5}}$

$=4(6^3)^{\frac{2}{3}}+(4^4)^{\frac{3}{4}}+2(3^5)^{\frac{1}{5}}$

$=4\times 6^{3\times\frac{2}{3}}+4^{4\times\frac{3}{4}}+2\times 3^{5\times\frac{1}{5}}$

$=4\times 6^2+4^3+2\times 3$

= 144 + 64 + 6 = 214

27. We have,

AE = AD [GIVEN] ...(1) and CE = BD [GIVEN] ...(2)

⇒ AE + CE = AD + BD [adding equation (1) & (2)]

⇒ AC = AB ...(3)

Now, in △AEB and △ADC,

AE = AD [given]

∠EAB = ∠DAC [common]

AB = AC [from (3)]

△AEB ≅ △ADC [by SAS]

28. Given: ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA. AC is a diagonal.

To Prove :

i. SR || AC and SR = $\frac{1}{2}$AC

ii. PQ = SR

iii. PQRS is a parallelogram

Proof :

i. In $\triangle$DAC,

As S is the mid-point of DA and R is the mid-point of DC

$\therefore$ SR || AC and SR = $\frac{1}{2}$AC . . . [Mid point theorem]

ii. In $\triangle$BAC,

As P is the mid-point of AB and Q is the mid-point of BC

$\therefore$ PQ || AC and PQ = $\frac{1}{2}$AC . . . [Mid point theorem]

But from (i) SR = $\frac{1}{2}$AC

$\therefore$ PQ = SR

iii. PQ || AC . . .[From (i)]

SR || AC . . .[From (i)]

$\therefore$ PQ || SR . . .[Two lines parallel to the same line are parallel to each other]

Similarly, PQ = SR . . .[From (ii)]

$\therefore$ PQRS is a parallelogram . . . [A quadrilateral is a parallelogram if a pair of opposite sides is parallel and is of equal length]

29. 2x – 3y + 7 = 0

⇒ 3y = 2x + 7

⇒ $y = \frac{2x+7}{3}$

Put x = 0, then $y = \frac{2(0)+7}{3} = \frac{7}{3}$

Put x = 1, then $y = \frac{2(1)+7}{3} = 3$

Put x = 2, then $y = \frac{2(2)+7}{3} = \frac{11}{3}$

Put x = 3, then $y = \frac{2(3)+7}{3} = \frac{13}{3}$

$\therefore (0, \frac{7}{3}), (1, 3), (2, \frac{11}{3})$ and $(3, \frac{13}{3})$ are the solutions of the equation 2x – 3y + 7 = 0.

30. Given that $\angle EPA = \angle DPB$

Adding $\angle$EPD on both sides, we get

$\angle \text{EPA} + \angle \text{EPD} = \angle \text{DPB} + \angle \text{EPD}$

$\Rightarrow \angle APD = \angle BPE$ ....(i)

Also given ,$\angle BAD = \angle ABE \Rightarrow \angle PAD = \angle PBE$ ....(ii)

Now in Δ APD and Δ BPE,

$\angle PAD = \angle PBE$. [from (ii)]

AP = PB [P is the mid-point of AB]

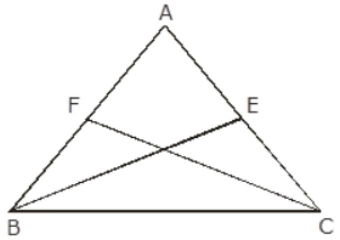
$\angle APD = \angle BPE$ [From (i)]

Hence ,by ASA congruency criteria;

$\Delta DAP \cong \Delta EBP$

⇒ AD = BE [By C.P.C.T.] Proved

OR



Given, ABC is an isosceles triangle

AB = AC

BE and CF are two medians

To prove: BE = CF

Proof: In$\triangle$BEC and $\triangle$CFB

CE = BF (SInce, AC = AB = $\frac{1}{2}$ AC = $\frac{1}{2}$ AB = CE = BF)

$\angle ECB = \angle FBC$ (Angle opposite to equal sides are equal)

BC = BC (Common)

Therefore By SAS theorem

$\triangle BEC \cong \triangle CFB$

BE = CF (By c.p.c.t.)

31. We know that

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

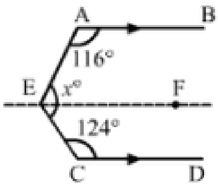
(Using Identity $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$)

$= (0)(x^2 + y^2 + z^2 - xy - yz - zx)$ ($\because x + y + z = 0$)

$= 0$

$\Rightarrow x^3 + y^3 + z^3 = 3xyz$.

Section D

32. 

Draw EF || AB || CD

Then, $\angle AEF + \angle CEF = x^\circ$

Now, EF || AB and AE is the transversal

$\therefore \angle AEF + \angle BAE = 180^\circ$ [Consecutive Interior Angles]

$\Rightarrow \angle AEF + 116 = 180$

$\Rightarrow \angle AEF = 64^\circ$

Again, EF || CD and CE is the transversal.

$\angle CEF + \angle ECD = 180^\circ$ [Consecutive Interior Angles]

$\Rightarrow \angle CEF + 124 = 180$

$\Rightarrow \angle CEF = 56^\circ$

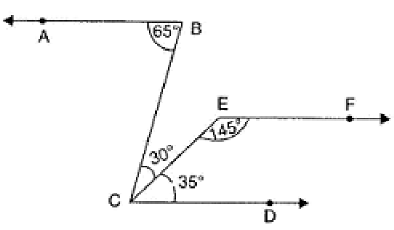
Therefore,

$x^\circ = \angle AEF + \angle CEF$

$x^\circ = (64 + 56)^\circ$

$x^\circ = 120^\circ$

OR



$\angle ABC = 65^o$

$\angle BCD = \angle BCE + \angle ECD = 30^o + 35^o = 65^o$

$\therefore \angle ABC = \angle BCD$

These angles form a pair of equal alternate angles

$\therefore$ AB || CD . . . (1)

$\angle FEC + \angle ECD = 145^o + 35^o = 180^o$

These angles are consecutive interior angles formed on the same side of the transversal.

$\therefore$ CD || EF . . . . (2)

AB || EF . . . [From (1) and (2)]

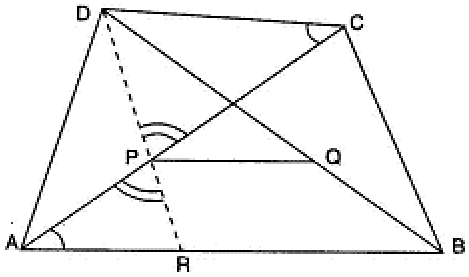
33. Given : ABCD is trapezium. P and Q are the mid-points of the diagonals AC and BD respectively.

To Prove :

i. PQ || AB or DC

ii. PQ = $\frac{1}{2}$ (AB – DC)

Construction : Join DP and produce DP to meet AB in R.



In $\triangle$APR and $\triangle$CPD,

$\angle$PAR = $\angle$PCD ....[Alternate angles]

$\angle$APR = $\angle$CPD ....[Vertically opp. angles]

AP = CP ...[Given]

$\therefore$ $\triangle$APR$\cong$ $\triangle$CPD . . .[By ASA axiom]

$\therefore$ PR = PD ....[c.p.c.t.]

and AR = CD ....[c.p.c.t.]

In $\triangle$ DRB,

As P and Q are the mid-points of DR and BD respectively.

$\triangle$ PQ || RB or AB or DC

and PQ = $\frac{1}{2}$RB = $\frac{1}{2}$(AB – AR) = $\frac{1}{2}$(AB – DC) .....[As AR = DC]

34. Suppose that the sides in metres are 6x, 7x and 8x.

Now, 6x + 7x + 8x = perimeter = 420

$\Rightarrow$ 21x = 420

$\Rightarrow$ $x = \frac{420}{21}$

$\Rightarrow$ x = 20

$\therefore$ The sides of the triangular field are $6 \times 20m, 7 \times 20m, 8 \times 20m,$ i.e., 120 m, 140 m and 160 m.

Now, s = Half the perimeter of triangular field.

$= \frac{1}{2} \times 420m = 210m$

Using Heron's formula,

Area of triangular field $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{210(210-120)(210-140)(210-160)}$

$= \sqrt{210 \times 90 \times 70 \times 50}$

$= \sqrt{66150000} = 8133.265m^2$

Hence, the area of the triangular field = 8133.265 m^2.

OR

Let x and y be the two lines of the right $\angle$

$\therefore$ AB = x cm, BC = y cm and AC = 10 cm

$\therefore$ By the given condition,

Perimeter = 24 cm

x + y +10 = 24 cm

Or x + y = 14 ... (I)

By Pythagoras theorem,

$x^2 + y^2 = (10)^2$ = 100 ... (II)

From (1), $(x+y)^2$ = $(14)^2$

Or $x^2 + y^2$ + 2xy = 196

$\therefore$100 + 2xy =196 [From (II)]

xy = $\frac{96}{2}$ = 48 sq cm .... (III)

Area of Δ ABC = $\frac{1}{2}xy$ sq cm

=$\frac{1}{2} \times 48$ sq cm

=24 sq cm.... (IV)

Again, we know that

$(x - y)^2 = (x+y)^2 - 4xy$

$=$ $(14)^2 - 4 \times 48$ [From (I) & (III)]

Or x - y = ± 2

(i) When, x-y = 2 and x+y = 14, then 2x = 16

or x = 8, y = 6

(ii) When, X – y = -2 and x + y = 14, then 2x = 12

Or x = 6, y = 8

Verification by using Heron's formula:

Sides are 6 cm, 8 cm and 10 cm

S = $\frac{24}{2}$ = 12 cm

Area of Δ ABC = $\sqrt{12\,(12-6)\,(12-8)\,(12-10)}$ sq cm

= $\sqrt{12 \times 6 \times 4 \times 2}$ sq cm

= 24 sq cm

Which is same as found in (IV)

Thus, the result is verified.

35. $x^3 - 2x^2 - x + 2$

We need to consider the factors of 2, which are $\pm 1, \pm 2$

Let us substitute 1 in the polynomial $x^3 - 2x^2 - x + 2$ to get

$(1)^3 - 2(1)^2 - (1) + 2 = 1 - 2 - 1 + 2 \ = 0$

Thus, according to factor theorem, we can conclude that $(x - 1)$ is a factor of the polynomial $x^3 - 2x^2 - x + 2$

Let us divide the polynomial $x^3 - 2x^2 - x + 2$ by $(x - 1)$,to get

$$
\begin{array}{r|l}
 & x^2 - x - 2 \\
x - 1 & x^3 - 2x^2 - x + 2 \\
 & \underline{x^3 - x^2} \\
 & \quad\;\; {}^{-}\quad {}^{+} \\
 & -x^2 - x \\
 & \underline{-x^2 + x} \\
 & \quad\;\; {}^{+}\quad {}^{-} \\
 & -2x + 2 \\
 & \underline{-2x + 2} \\
 & \quad\;\; {}^{+}\quad {}^{-} \\
 & 0
\end{array}
$$

$x^3 - 2x^2 - x + 2 = (x - 1)\left(x^2 - x - 2\right).$

$= (x - 1)\left(x^2 + x - 2x - 2\right)$

$= (x - 1)\left[x\,(x + 1) - 2\,(x + 1)\right]$

$= (x - 1)\,(x - 2)\,(x + 1).$

Therefore, we can conclude that on factorizing the polynomial $x^3 - 2x^2 - x + 2$,we get $(x - 1)\,(x - 2)\,(x + 1)$

Section E

36. Diameter of cone = 40 cm

⇒ Radius of cone (r) = $\frac{40}{2}$

= 20 cm

= $\frac{20}{100}$ m

= 0.2 m

Height of cone (h) = 1 m

Slant height of cone $(l) = \sqrt{r^2 + h^2}$

= $\sqrt{(0.2)^2 + (1)^2}$

= $\sqrt{1.04}$ m

Curved surface area of cone = $\pi r l$

= 3.14 × 0.2 × $\sqrt{1.04}$

$= 0.64056\ m^2$

$\because$ Cost of painting $1m^2$ of a cone = Rs.12

$\therefore$ Cost of painting $0.64056m^2$ of a cone = 12×0.64056 = Rs. 7.68672

$\therefore$ Cost of painting of 50 such cones = 50×7.68672 = Rs. 384.34 (approx.)

37. i. Expenses in 2009-10 = 9160 Million

Expenses in 2010-11 = 10300 Million

Total expenses from 2009 to 2011

= 9160 + 10300

= 19460 Million

ii. Expenses in 2009-10 = 9160 Million

Expenses in 2010-11 = 10300 Million

Thus percentage of no of expenses in 2009-10 over the expenses in 2010-11

$= \frac{9160}{10300} \times 100$

= 88.93%

iii. The minimum expenses (in 2007-08) = 5.4 Million

The maximum expenses (in 2010-11) = 10300 Million

Thus percentage of no of minimum expenses over the maximum expenses

$= \frac{5.4}{10300} \times 100$

= 0.052%

OR

The expenses in 2010-11 = 10300 Million

The expenses in 2006-09 = 9060 Million

The difference = 10300 - 9060 Million

= 1240 Million

38. i. We know that angle in the semicircle = 90^o

Here QR is a diameter of circle and $\angle$QPR is angle in semicircle.

Hence $\angle QPR = 90^o$

ii. $\angle QPR = 90^o$

$\Rightarrow QR^2 = PQ^2 + PR^2$

$\Rightarrow QR^2 = 8^2 + 6^2$

$\Rightarrow QR = \sqrt{64 + 36}$

$\Rightarrow$ QR = 10 m

iii. Measure of $\angle QSR = 90^o$

Angles in the same segment are equal. $\angle$QSR and $\angle$QPR are in the same segment.

OR

Area $\Delta PQR = \frac{1}{2} \times PQ \times PR$

$\Rightarrow$ Area $\Delta PQR = \frac{1}{2} \times 8 \times 6$ = 24 sqm